SEMSETRAL EXAMINATION
M. Math II YEAR, I SEMSTER, 2016-17
FOURIER ANALYSIS

Max. marks: 100 Time limit: 3hrs

1. Let $T : \mathbb{R}^k \to \mathbb{R}^k$ be a non-singular linear transformation. Let $f \in L^1(\mathbb{R}^k)$ and $g(x) = f(Tx)\ \forall x \in \mathbb{R}^k$. Compute that Fourier transform of g in terms of the Fourier transform of f. [10]

2. Let $f \in L^1(\mathbb{R})$, μ be a Borel probability measure on $\mathbb{R}$ and
$f(x) = \int f(x-y)d\mu(y)$ a.e..
If $\int f(x)dx \neq 0$ show that $\mu\{0\} = 1$. [15]
Hint: take Fourier transforms and show that $\int\{1 - \cos tx\}d\mu(x) = 0$ for a sufficiently large set of $t's$.

3. Let $f \in L^1([-\pi,\pi])$ and $\hat{f}(n) \neq 0\ \forall n \in \mathbb{Z}$. Show that $\{f * g : g \in L^1([-\pi,\pi])\}$ is dense in $L^1([-\pi,\pi])$. [30]

Hint: let h be a continuous periodic function and
$h_N(x) = \sum_{j=-N}^{N} \frac{\hat{h}(j)}{\hat{f}(j)}(1 - \frac{|j|}{N+1})e^{ijx}$.
Use Fejer's Theorem to prove that $f * h_N \to h$ in $L^1([-\pi,\pi])$.

4. Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$. Show that the Hilbert transform g of f is given by
$g(x) = -\frac{x^2}{4} + \frac{\pi x}{2} - \frac{\pi^2}{6}$ for $0 \leq x \leq 2\pi$. [25]

5.

a) Is $\sqrt{2}I_{(0.\frac{1}{2})}$ a scaling function? Justify. [5]

b) Is $I_{[0,1)} - I_{[1,2]}$ a (continuum) wavelet?. Justify. [15]